Fundamentos de Maquinas Eléctricas

CHAPTER 7

Active, Reactive, and Apparent Power

7.0 Introduction

The concept of active, reactive, and apparent power plays a major role in electric power technology. In effect, the transmission of electrical energy and the behavior of ac machines are often easier to understand by working with power, rather than dealing with voltages and currents. The reader is therefore encouraged to pay particular attention to this chapter.

The terms *active, reactive,* and *apparent power* apply to steady-state alternating current circuits in which the voltages and currents are sinusoidal. They cannot be used to describe transient-state behavior, nor can we apply them to dc circuits.

Our study begins with an analysis of the instantaneous power in an ac circuit. We then go on to define the meaning of active and reactive power and how to identify sources and loads. This is followed by a definition of apparent power, *power factor,* and the *power triangle.* We then show how ac circuits can be solved using these power concepts. In conclusion, vector notation is used to determine the active and reactive power in an ac circuit.

7.1 Instantaneous power

The instantaneous power supplied to a device is simply the product of the instantaneous voltage across its terminals times the instantaneous current that flows through it.

Instantaneous power is always expressed in watts, irrespective of the type of circuit used. The instantaneous power may be positive or negative. A positive value means that power flows into the device. Conversely, a negative value indicates that power is flowing out of the device.

Example 7-1

A sinusoidal voltage having a peak value of 162 V and a frequency of 60 Hz is applied to the terminals of an ac motor. The resulting current has a peak value of 7.5 A and lags 50° behind the voltage.

a. Express the voltage and current in terms of the electrical angle ϕ.
b. Calculate the value of the instantaneous voltage and current at an angle of 120°.

c. Calculate the value of the instantaneous power at 120°.

d. Plot the curve of the instantaneous power delivered to the motor.

Solution

a. Let us assume that the voltage starts at zero and increases positively with time. We can therefore write

$$e = E_{\mathrm{m}} \sin \phi = 162 \sin \phi$$

The current lags behind the voltage by an angle $\theta = 50°$, consequently, we can write

$$i = I_{\mathrm{m}} \sin (\phi - \theta) = 7.5 \sin (\phi - 50°)$$

b. At $\phi = 120°$ we have

$$e = 162 \sin 120° = 162 \times 0.866$$
$$= 140.3 \text{ V}$$
$$i = 7.5 \sin (120° - 50°) = 7.5 \sin 70°$$
$$= 7.5 \times 0.94$$
$$= 7.05 \text{ A}$$

c. The instantaneous power at 120° is

$$p = ei = 140.3 \times 7.05 = +989 \text{ W}$$

Because the power is positive, it flows at this instant into the motor.

d. In order to plot the curve of instantaneous power, we repeat procedures (b) and (c) above for angles ranging from $\phi = 0$ to $\phi = 360°$. Table 7A lists part of the data used.

TABLE 7A VALUES OF e, i, AND p USED TO PLOT FIG. 7.1

Angle ϕ degrees	Voltage $162 \sin \phi$ volts	Current $7.5 \sin (\phi - 50°)$ amperes	Power p watts
0	0	−5.75	0
25	68.5	−3.17	−218
50	124.1	0	0
75	156.5	3.17	497
115	146.8	6.8	1000
155	68.5	7.25	497
180	0	5.75	0
205	−68.5	3.17	−218
230	−124.1	0	0

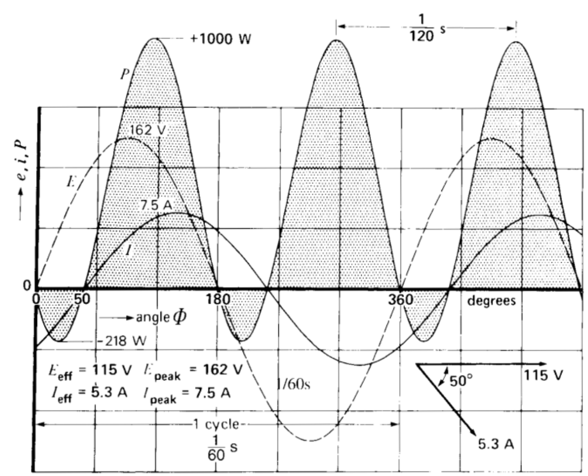

Figure 7.1
Instantaneous voltage, current and power in an ac circuit. (See Example 7-1.)

The voltage, current, and instantaneous power are plotted in Fig. 7.1. The power attains a positive peak of +1000 W and a negative peak of −218 W. The negative power means that power is actually flowing from the load (motor) to the source. This occurs during the intervals 0 − 50°, 180° − 230°, and 360° − 410°. Although a power flow from a device considered to be a load to a device considered to be a source may seem to be impossible, it happens often in ac circuits. The reason is given in the sections that follow.

We also note that the positive peaks occur at intervals of 1/120 s. This means that the frequency of the power cycle is 120 Hz, which is twice the frequency of the voltage and current that produce the power. Again, this phenomenon is quite normal: the frequency of ac power flow is always twice the line frequency.

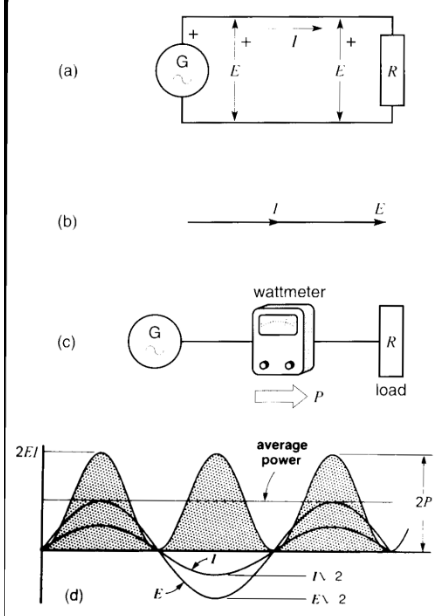

Figure 7.2

a. An ac voltage E produces an ac current I in this resistive circuit.
b. Phasors E and I are in phase.
c. A wattmeter indicates EI watts.
d. The active power is composed of a series of positive power pulses.

7.2 Active power*

The simple ac circuit of Fig. 7.2a consists of a resistor connected to an ac generator. The effective voltage and current are designated E and I, respectively, and as we would expect in a resistive circuit, phasors E and I are in phase (Fig. 7.2b). If we connect a wattmeter (Fig. 7.3) into the line, it will give a reading $P = EI$ watts (Fig. 7.2c).

To get a better picture of what goes on in such a circuit, we have drawn the sinusoidal curves of E and I (Fig. 7.2d). The peak values are respectively $\sqrt{2}E$ volts and $\sqrt{2}I$ amperes because, as stated previously, E and I are *effective* values. By multiplying the instantaneous values of voltage and current as we did in Section 7.1, we obtain the *instantaneous* power in watts.

* Many persons refer to active power as *real power* or *true power*, considering it to be more descriptive. In this book we use the term *active power*, because it conforms to the IEEE designation.

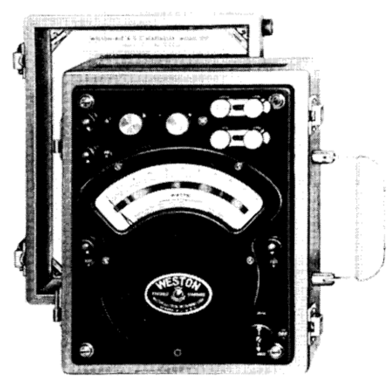

Figure 7.3
Example of a high-precision wattmeter rated 50 V, 100 V, 200 V; 1 A, 5 A. The scale ranges from 0-50 W to 0-1000 W.
(*Courtesy of Weston Instruments*)

The power wave consists of a series of positive pulses that vary from zero to a maximum value of $(\sqrt{2}E) \times (\sqrt{2}I) = 2EI = 2P$ watts. The fact that power is always positive reveals that it always flows from the generator to the resistor. This is one of the basic properties of what is called *active power:* although it pulsates between zero and maximum, it *never* changes direction. The direction of power flow is shown by an arrow P (Fig. 7.2c).

The average power is clearly midway between $2P$ and zero, and so its value is $P = 2EI/2 = EI$ watts. That is precisely the power indicated by the wattmeter.

The two conductors leading to the resistor in Fig. 7.2a carry the active power. However, unlike current flow, power does not flow down one conductor and return by the other. Power flows over *both* conductors and, consequently, as far as power is concerned, we can replace the conductors by a single line, as shown in Fig. 7.2c.

In general, the line represents any transmission line connecting two devices, irrespective of the number of conductors it may have.

The generator is an *active source* and the resistor is an *active load*. The symbol for active power is P and the unit is the watt (W). The kilowatt (kW) and megawatt (MW) are frequently used multiples of the watt.

7.3 Reactive power

The circuit of Fig. 7.4a is identical to the resistive circuit (Fig. 7.2a) except that the resistor is now replaced by a reactor X_L. As a result, current I lags 90° behind the voltage E (Fig. 7.4b).

To see what really goes on in such a circuit, we have drawn the waveforms for E and I and, by again multiplying their instantaneous values, we obtain the curve of instantaneous power (Fig. 7.4c). This power p consists of a series of identical positive and negative pulses. The positive waves correspond to instantaneous power delivered by the generator to the reactor and the negative waves represent instantaneous power delivered from the reactor to the generator. The duration of each wave corresponds to one quarter of a cycle of the line frequency. The

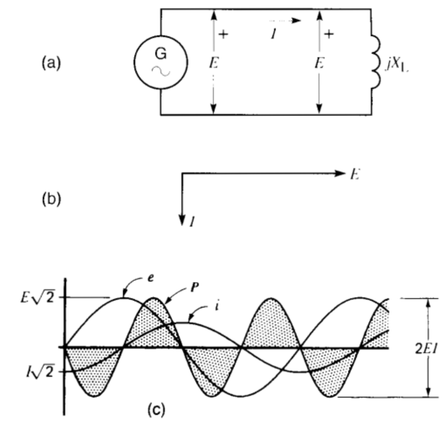

Figure 7.4
a. An ac voltage E produces an ac current I in this inductive circuit.
b. Phasor I lags 90° behind E.
c. Reactive power consists of a series of positive and negative power pulses.

frequency of the power wave is therefore again twice the line frequency.

Power that surges back and forth in this manner is called *reactive power* (symbol Q), to distinguish it from the unidirectional active power mentioned before. The reactive power in Fig. 7.4 is also given by the product EI. However, to distinguish this power from active power, another unit is used—the **var.** Its multiples are the kilovar (kvar) and megavar (Mvar).

Special instruments, called *varmeters*, are available to measure the reactive power in a circuit (Fig. 7.5). A varmeter registers the product of the effective line voltage E times the effective line current I times sin θ (where θ is the phase angle between E and I). A reading is only obtained when E and I are out of phase; if they are exactly in phase (or exactly 180° out of phase), the varmeter reads zero.

Returning to Fig. 7.4, the dotted area under each pulse is the energy, in joules, transported in one direction or the other. Clearly, the energy is delivered in a continuous series of pulses of very short duration, every positive pulse being followed by a

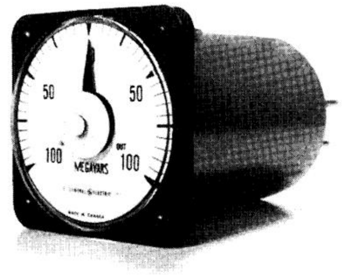

Figure 7.5
Varmeter with a zero-center scale. It indicates positive or negative reactive power flow up to 100 Mvars.

negative pulse. The energy flows back and forth between the generator and the inductor without ever being used up.

What is the reason for these positive and negative energy surges? The energy flows back and forth because magnetic energy is alternately being stored up and released by the reactor. Thus, when the power is positive, the magnetic field is building up inside the coil. A moment later when the power is negative, the energy in the magnetic field is decreasing and flowing back to the source.

We now have an explanation for the brief negative power pulses in Fig. 7.1. In effect, they represent magnetic energy, previously stored up in the motor windings, that is being returned to the source.

7.4 Definition of a reactive load and reactive source

Reactive power involves real power that oscillates back and forth between two devices over a transmission line. Consequently, it is impossible to say whether the power originates at one end of the line or the other. Nevertheless, it is useful to assume that some devices generate reactive power while others absorb it. In other words, some devices behave like *reactive sources* and others like *reactive loads*.

By definition*, a reactor is considered to be a reactive load that absorbs reactive power.

Example 7-2

A reactor having an inductive reactance of 4 Ω is connected to the terminals of a 120 V ac generator (Fig. 7.6a).

a. Calculate the value of the current in the reactor
b. Calculate the power associated with the reactor
c. Calculate the power associated with the ac generator
d. Draw the phasor diagram for the circuit

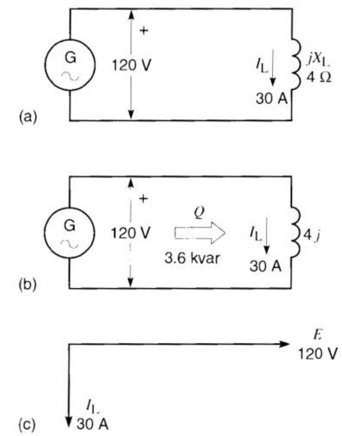

Figure 7.6
See Example 7.2.

Solution

a. Current in the circuit:

$$I_L = \frac{E}{X_L} = \frac{120\ \text{V}}{4\ \Omega} = 30\ \text{A}$$

b. Power associated with the reactor:

$$Q = EI = 120 \times 30 = 3600\ \text{var} = 3.6\ \text{kvar}$$

*This definition is in agreement with IEEE and IEC conventions.

This reactive power is absorbed by the reactor.

c. Because the reactor absorbs 3.6 kvar of reactance power, the ac generator must be supplying it. Consequently, the generator is a source of reactive power: it delivers 3.6 kvar. The reactive power Q flows therefore in the direction shown (Fig. 7.6b).
d. The phasor diagram is shown in Fig. 7.6c. Current I_L lags 90° behind voltage E.

This phasor diagram applies to the reactive load (the reactor) and the reactive source (the ac generator) as well as the line connecting them.

7.5 The capacitor and reactive power

Suppose now that we add a capacitor having a reactance of 4 Ω to the circuit of Fig. 7.6. This yields the circuit of Fig. 7.7a. The current I_c drawn by the capacitor is $I_c = 120\ \text{V}/4\ \Omega = 30\ \text{A}$ and, as we would expect, it leads the voltage by 90° (Fig. 7.7b).

The vector sum of I_L and I_c is zero and so the ac generator is no longer supplying any power at all to the circuit. However, the current in the reactor has not changed; consequently, it continues to absorb 30 A × 120 V = 3.6 kvar of reactive power.

Where is this reactive power coming from? It can only come from the capacitor, which acts as a source of reactive power. The reactive power delivered by the capacitor is equal to the current it carries times the voltage across its terminals, namely

$$Q = EI_c = 120\ \text{V} \times 30\ \text{A} = 3600\ \text{var} = 3.6\ \text{kvar}$$

The reactive power delivered by the capacitor is expressed in vars or kilovars. Reactive power Q now flows from the capacitor to the reactor.

We have arrived at a very important conclusion: a *capacitor is a source of reactive power.* It acts as a reactive power source whenever it is part of a sine-wave-based, steady-state circuit.

Let us take another step and remove the reactor from the circuit in Fig. 7.7a, yielding the circuit of Fig. 7.8a. The capacitor is now alone, connected to the terminals of the ac generator. It still carries a current of 30 A, leading the voltage E by 90° (Fig. 7.8b). Consequently, the capacitor still acts as a source of reactive power, delivering 3.6 kvar. Where does this power go? The answer is that the capacitor delivers reactive power to the very generator to

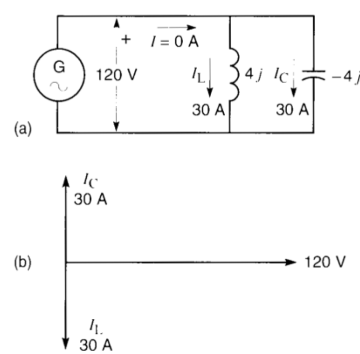

Figure 7.7
See Example 7.3.

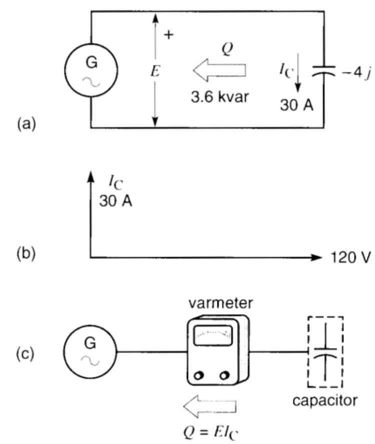

Figure 7.8
a. Capacitor connected to an ac source.
b. Phasor I_c leads E by 90°.
c. Reactive power flows from the capacitor to the generator.

which it is connected! For most people, this takes a little time to accept. How, we may ask, can a passive device like a capacitor possibly produce any power? The answer is that reactive power really represents energy that, like a pendulum, swings back and forth without ever doing any useful work. The capacitor acts as a temporary energy-storing device repeatedly accepting energy for brief periods and releasing it again. However, instead of storing magnetic energy as a reactor does, a capacitor stores electrostatic energy (see Section 2.14).

If we connect a varmeter into the circuit (Fig. 7.8c), it will give a negative reading of $EI = -3600$ var, showing that reactive power is indeed flowing from the capacitor to the generator. The generator is now behaving like reactive load, but we sometimes prefer to call it a *receiver* of reactive power, which, of course, amounts to the same thing. In summary, a capacitive reactance *always* generates reactive power.

Example 7-3

An ac generator G is connected to a group of *R, L, C* circuit elements (Fig. 7.9). The respective elements carry the currents shown. Calculate the active and reactive power associated with the generator.

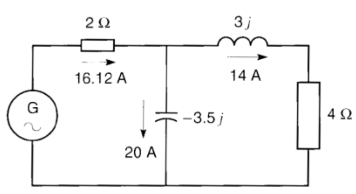

Figure 7.9
See Example 7.3.

Solution

The two resistors absorb active power given by

$$P = I^2R = (14^2 \times 4) + (16.12^2 \times 2) = 784 + 520 = 1304 \text{ W}$$

The 3 Ω reactor absorbs reactive power:
$Q_L = I^2X_L = 14^2 \times 3 = 588$ var

The 3.5 Ω capacitor generates reactive power:
$Q_c = I^2X_c = 20^2 \times 3.5 = 1400$ var

The *R, L, C* circuit generates a net reactive power of $1400 - 588 = 812$ var

This reactive power must be absorbed by the generator; hence, as far as reactive power is concerned the generator acts as a load.

The active power absorbed by the resistors must be supplied by the generator; hence it is a source of active power = 1304 W.

In conclusion, the ac generator is a source of active power (1304 W) and a receiver of reactive power (812 var).

7.6 Distinction between active and reactive power

There is a basic difference between active and reactive power, and perhaps the most important thing to remember is that the one cannot be converted into the other. Active and reactive powers function independently of each other and, consequently, they can be treated as separate quantities in electric circuits.

Both place a burden on the transmission line that carries them, but, whereas active power eventually produces a tangible result (heat, mechanical power, light, etc.), reactive power only represents power that oscillates back and forth.

All ac inductive devices such as magnets, transformers, ballasts, and induction motors, absorb reactive power because one component of the current they draw lags 90° behind the voltage. The reactive power plays a very important role because it produces the ac magnetic field in these devices.

A building, shopping center, or city may be considered to be a huge active/reactive load connected to an electric utility system. Such load centers contain thousands of induction motors and other electromagnetic devices that draw both reactive power (to sustain their magnetic fields) and active power (to do the useful work).

This leads us to the study of loads that absorb both active and reactive power.

7.7 Combined active and reactive loads: apparent power

Loads that absorb both active power P and reactive power Q may be considered to be made up of a resistance and an inductive reactance. Consider, for example, the circuit of Fig. 7.10a in which a resistor and reactor are connected to a source G. The resistor draws a current I_p, while the reactor draws a current I_q.

According to our definitions, the resistor is an active load while the reactor is a reactive load. Consequently, I_p is in phase with E while I_q lags 90° behind. The phasor diagram (Fig. 7.10b) shows that the resultant line current I lags behind E by an angle θ. Furthermore, the magnitude of I is given by

$$I = \sqrt{I_p^2 + I_q^2}$$

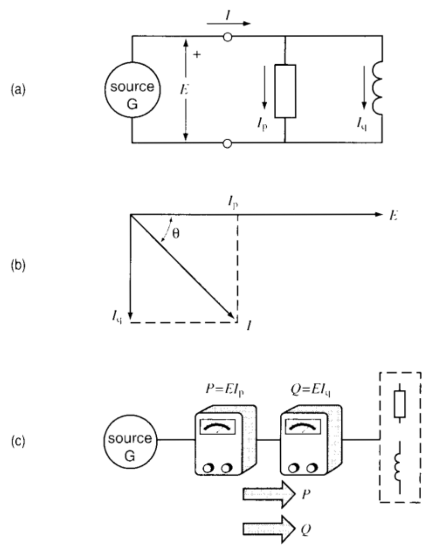

Figure 7.10
a. Circuit consisting of a source feeding an active and reactive load.
b. Phasor diagram of the voltage and currents.
c. Active and reactive power flow from source to load.

The active and reactive power components P and Q both flow in the same direction, as shown by the arrows in Fig. 7.10c. If we connect a wattmeter and a varmeter into the circuit, the readings will both be positive, indicating $P = EI_p$ watts and $Q = EI_q$ vars, respectively.

Furthermore, if we connect an ammeter into the line, it will indicate a current of I amperes. As a result, we are inclined to believe that the power supplied to the load is equal to EI watts. But this is obviously incorrect because the power is composed of an active component (watts) and a reactive component (vars). For this reason the product EI is called *apparent power.* The symbol for apparent power is S.

Apparent power is expressed neither in watts nor in vars, but in *voltamperes*. Multiples are the kilovoltampere (kVA) and megavoltampere (MVA).

7.8 Relationship between *P, Q,* and *S*

Consider the single-phase circuit of Fig. 7.11a composed of a source, a load, and appropriate meters. Let us assume that

- the voltmeter indicates E volts
- the ammeter indicates I amperes
- the wattmeter indicates $+P$ watts
- the varmeter indicates $+Q$ vars

Knowing that P and Q are positive, we know that the load absorbs both active and reactive power. Consequently, the line current I lags behind E_{ab} by an angle θ.

Current I can be decomposed into two components I_p and I_q, respectively in phase, and in quadrature, with phasor E (Fig. 7.11b). The numerical values of I_p and I_q can be found directly from the instrument readings

$$I_p = P/E \tag{7.1}$$

$$I_q = Q/E \tag{7.2}$$

Furthermore, the apparent power S transmitted over the line is given by $S = EI$, from which

$$I = S/E \tag{7.3}$$

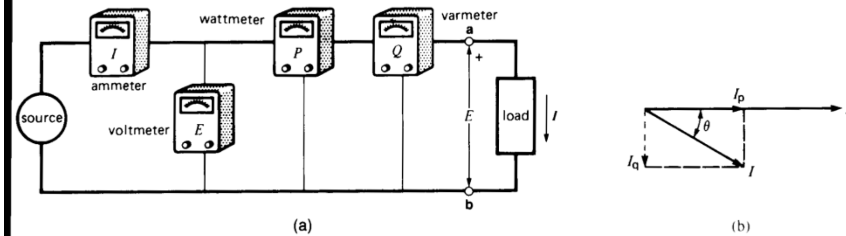

Figure 7.11
a. Instruments used to measure E, I, P, and Q in a circuit.
b. The phasor diagram can be deduced from the instrument readings.

Referring to the phasor diagram (Fig. 7.11b), it is obvious that

$$I^2 = I_p^2 + I_q^2$$

Consequently,

$$\left[\frac{S}{E}\right]^2 = \left[\frac{P}{E}\right]^2 + \left[\frac{Q}{E}\right]^2$$

That is,

$$S^2 = P^2 + Q^2 \qquad (7.4)$$

in which

S = apparent power [VA]
P = active power [W]
Q = reactive power [var]

We can also calculate the value of the angle θ because the tangent of θ is obviously equal to I_q/I_p. Thus, we have

$$\theta = \arctan I_q/I_p = \arctan Q/P \qquad (7.5)$$

Example 7-4

An alternating-current motor absorbs 40 kW of active power and 30 kvar of reactive power. Calculate the apparent power supplied to the motor.

Solution

$$S = \sqrt{P^2 + Q^2} \qquad (7.4)$$
$$= \sqrt{40^2 + 30^2}$$
$$= 50 \text{ kVA}$$

Example 7-5

A wattmeter and varmeter are connected into a 120 V single-phase line that feeds an ac motor. They respectively indicate 1800 W and 960 var.

Calculate

a. The in-phase and quadrature components I_p and I_q
b. The line current I
c. The apparent power supplied by the source
d. The phase angle between the line voltage and line current

Solution

Referring to Fig. 7.11, where the load is now a motor, we have

a. $$I_p = P/E = 1800/120 = 15 \text{ A} \qquad (7.1)$$
$$I_q = Q/E = 960/120 = 8 \text{ A} \qquad (7.2)$$

b. From the phasor diagram we have

$$I = \sqrt{I_p^2 + I_q^2} = \sqrt{15^2 + 8^2}$$
$$= 17 \text{ A}$$

c. The apparent power is

$$S = EI = 120 \times 17 = 2040 \text{ VA}$$

d. The phase angle θ between E and I is

$$\theta = \arctan Q/P = \arctan 960/1800$$
$$= 28.1°$$

Example 7-6

A voltmeter and ammeter connected into the inductive circuit of Fig. 7.4a give readings of 140 V and 20 A, respectively.

Calculate

a. The apparent power of the load
b. The reactive power of the load
c. The active power of the load

Solution

a. The apparent power is

$$S = EI = 140 \times 20$$
$$= 2800 \text{ VA} = 2.8 \text{ kVA}$$

b. The reactive power is

$$Q = EI = 140 \times 20$$
$$= 2800 \text{ var} = 2.8 \text{ kvar}$$

If a varmeter were connected into the circuit, it would give a reading of 2800 var.

c. The active power is zero.

If a wattmeter were connected into the circuit, it would read zero.

To recapitulate, the apparent power is 2800 VA, but because the current is 90° out of phase with the voltage, it is also equal to 2800 var.

7.9 Power factor

The power factor of an alternating-current device or circuit is the ratio of the active power P to the apparent power S. It is given by the equation

$$\text{power factor} = P/S \qquad (7.6)$$

where

P = active power delivered or absorbed by the circuit or device [W]

S = apparent power of the circuit or device [VA]

Power factor is expressed as a simple number, or as a percentage.

Because the active power P can never exceed the apparent power S, it follows that the power factor can never be greater than unity (or 100 percent). The power factor of a resistor is 100 percent because the apparent power it draws is equal to the active power. On the other hand, the power factor of an ideal coil having no resistance is zero, because it does not consume any active power.

To sum up, the power factor of a circuit or device is simply a way of stating what fraction of its apparent power is real, or active, power.

In a single-phase circuit the power factor is also a measure of the phase angle θ between the voltage and current. Thus, referring to Fig. 7.11,

$$\text{power factor} = P/S$$
$$= EI_p/EI$$
$$= I_p/I$$
$$= \cos\theta$$

Consequently,

$$\text{power factor} = \cos\theta = P/S \qquad (7.7)$$

where

power factor = power factor of a single-phase circuit or device

θ = phase angle between the voltage and current

If we know the power factor, we automatically know the cosine of the angle between E and I and, hence, we can calculate the angle. The power factor is said to be *lagging* if the current lags behind the voltage. Conversely, the power factor is said to be *leading* if the current leads the voltage.

Example 7-7

Calculate the power factor of the motor in Example 7-5 and the phase angle between the line voltage and line current.

Solution

$$\text{power factor} = P/S$$
$$= 1800/2040$$
$$= 0.882 \text{ or } 88.2\% \text{ (lagging)}$$
$$\cos\theta = 0.882$$
$$\text{therefore, } \theta = 28.1°$$

Example 7-8

A single-phase motor draws a current of 5 A from a 120 V, 60 Hz line. The power factor of the motor is 65 percent.

Calculate

a. The active power absorbed by the motor
b. The reactive power supplied by the line

Solution

a. The apparent power drawn by the motor is

$$S_m = EI = 120 \times 5 = 600 \text{ VA}$$

The active power absorbed by the motor is

$$P_m = S_m \cos \theta \qquad (7.7)$$
$$= 600 \times 0.65 = 390 \text{ W}$$

b. The reactive power absorbed by the motor is

$$Q_m = \sqrt{S_m^2 - P_m^2} \qquad (7.4)$$
$$= \sqrt{600^2 - 390^2}$$
$$= 456 \text{ var}$$

Note that the motor draws even more reactive power from the line than active power. This burdens the line with a relatively large amount of nonproductive power.

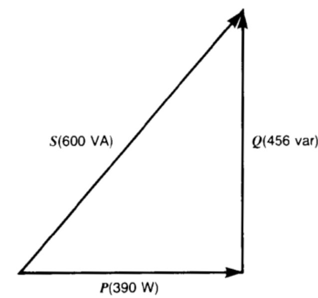

Figure 7.12
Power triangle of a motor. See Example 7-8.

7.10 Power triangle

The $S^2 = P^2 + Q^2$ relationship expressed by Eq. 7.4, brings to mind a right-angle triangle. Thus, we can show the relationship between S, P, and Q graphically by means of a *power triangle*. According to convention, the following rules apply:

1. Active power P absorbed by a circuit or device is considered to be positive and is drawn horizontally to the right
2. Active power P that is delivered by a circuit or device is considered to be negative and is drawn horizontally to the left
3. Reactive power Q absorbed by a circuit or device is considered to be positive and is drawn vertically upwards
4. Reactive power Q that is delivered by a circuit or device is considered to be negative and is drawn vertically downwards

The power triangle for Example 7-8 is shown in Fig. 7.12 in accordance with these rules. The power components S, P, and Q look like phasors, but they are not. However, we can think of them as convenient vectors. The concept of the power triangle is useful when solving ac circuits that comprise several active and reactive power components.

7.11 Further aspects of sources and loads

Let us consider Fig. 7.13a in which a resistor and capacitor are connected to a source. The circuit is similar to Fig. 7.10 except that the capacitor is a reactive source. As a result, reactive power flows from the capacitor to the source G while active power flows from the source G to the resistor. The active and reactive power components therefore flow in opposite directions over the transmission line. A wattmeter connected into the circuit will give a positive reading $P = EI_p$ watts, but a varmeter will give a negative reading $Q = EI_q$. The source G *delivers* active power P but *receives* reactive power Q. Thus, G is simultaneously an active source *and* a reactive load.